

Problem Set: Set Theory I

- Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$ and $C = \{6, 8\}$. Find following:
 - $A \cup B$
 - $A \cap B$
 - $A \cap B^C$
 - $B - A$
 - $C - B$
 - $A \cap C$
- Let $A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4\}$ and $C = \{a, b, 1, 2\}$. Show that:
 - Distributivity: $(A \cap C) \cup (B \cap C) = (A \cup B) \cap C$
 - Associativity: $(A \cap B) \cap C = A \cap (B \cap C)$
 - De Morgan Laws: $C - (A \cup B) = (C - A) \cap (C - B)$
- Determine which of the following formulas are true. If any formula is false, find a counterexample to demonstrate this using a Venn diagram.
 - $A \setminus B = B \setminus A$
 - $A \subseteq B \iff A \cap B = A$
 - $A \cup B = A \cup C \implies B = C$
 - $A \subseteq B \iff A \cup B = B$
 - $A \cap B = A \cap C \implies B = C$
 - $A \setminus (B \setminus C) = (A \setminus B) \setminus C$
- Explain in words why it is true that for any sets A, B, C :
 - $(A \cup B) \cup C = A \cup (B \cup C)$
 - $(A \cap B) \cap C = A \cap (B \cap C)$
 - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(d) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

5. Find the interior point(s) and the boundary points(s) of the set $\{x : 1 \leq x \leq 5\}$.
6. Why does every set in \mathbb{R} that is nonempty, closed, and bounded have a greatest member?
7. Which of the following sets are open, closed, or neither?

(a) $D = \{x \in \mathbb{R}^1 : x = 2 \text{ or } 3 < x < 4\}$

(b) $A = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 1\}$

(c) $B = \{(x, y) \in \mathbb{R}^2 : x^2 < y < 1\}$

(d) $C = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y < 1\}$

(e) universal set

8. Sketch the following functions:

(a) $f(x) = 2$

(b) $f(x) = 3x - 1$

(c) $f(x) = x^2 + 2x + 1$

(d) $f(x) = (x - 3)^{-1}$

(e) $f(x) = |2x - 2|$

(f) $f(x) = e^{2x}$

(g) $f(x) = -\sqrt{x}$

9. Which of the following functions is injective, bijective, or surjective?

(a) $a(x) = 2x + 1$

(b) $b(x) = x^2$

(c) $c(x) = \ln x$

(d) $d(x) = e^x$